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The magnetic field effect on the screened Coulomb interaction between two test charges in the presence of a vacuum–solid interface

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Abstract. In this paper, the screened Coulomb interaction between two charged particles near a vacuum–solid interface is calculated in the presence of a quantising magnetic field which is perpendicular to the surface. The infinite-potential-barrier model is assumed and a background dielectric constant is included in the bulk dielectric function to simulate different types of material. Closed-form analytical expressions are derived when both charges are on the same side of the surface and also when one is in the vacuum region and the other is embedded within the plasma. In the latter case, numerical results are obtained in the quantum strong-field limit. On the assumption that the test charges are both far away from the surface, and on opposite sides of it, asymptotic expressions are obtained for the electrostatic Coulomb interaction. When the source is in the material medium and the field point is outside, the interaction potential decays at large distances according to a power law. However, when the source is outside and the field point is inside, the potential has a Friedel–Kohn wiggle.

A preponderance of solid state phenomena is a consequence of the Coulomb interaction and involves the application of electric and magnetic fields to solid surfaces from where they communicate with the bulk. Therefore it is of some considerable interest to examine how the interaction potential between internal and external probes, such as electrons, atoms and ions, is modified by the presence of a surface. In this paper, we derive expressions for the Coulomb interaction when a strong magnetic field is applied perpendicular to the boundary plane between a quantum electron plasma and a vacuum region (figure 1). We include the electron–electron interactions by means of the random-phase approximation (RPA), and the infinite-barrier model (IBM) is employed to describe the boundary surface. We are particularly concerned with how the screening of the interaction between two test charges is affected in the surface region and how this interaction approaches the well known bulk behaviour as the charges recede into the plasma. For simplicity, we have limited our calculations to the so-called high-field quantum limit (HFQL), where all electrons are condensed into the highly degenerate lowest Landau level. This situation is at present realisable only for a few systems, such

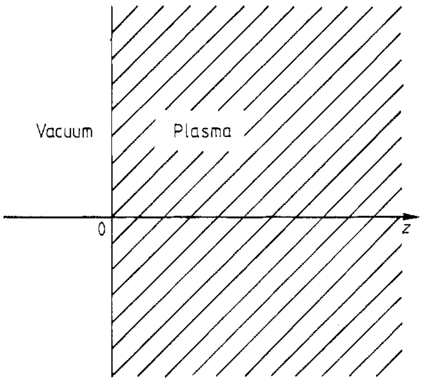

Figure 1. Geometry for the IBM.

as doped narrow-gap semiconductors, and our results might apply to Shubnikov–de Haas or cyclotron resonance in such substances. To apply them to data that are at present available, however, the calculations will have to be extended to lower magnetic field strengths. Nevertheless, we feel that the present calculation represents an important first step.

In this paper, the magnetic field is applied in the z direction perpendicular to the boundary plane of the quantum plasma of background dielectric constant ε_0 occupying the half-space $z > 0$ (see figure 1). The electron–electron interactions are treated in the RPA, and the IBM is employed to describe the boundary condition of specular reflection for electrons at $z = 0$.

Over the years, several researchers have calculated the screened Coulomb interaction between two charged particles both of which are situated either inside the quantum plasma or in the vacuum region outside [1–7]. The present work is concerned with magnetic field effects on the shielded potential between an impurity at $\boldsymbol{r}_0 = (0, 0, z_0)$ and a field point on the opposite side of a planar surface. Quantum interference effects due to electron wavefunction reflections at the infinite potential barrier will be neglected. Our method of calculation is based on the use of the inverse dielectric function for a semi-infinite quantum plasma which has been determined in the literature for the diagonal approximation [3]. In this paper, we study the effect on the Coulomb interaction due to a magnetic field which was not discussed in [1]. In this context, [8–10] are suitable references for the HFQL.

If the potential $U(\boldsymbol{r})$ is impressed on a system described by the inverse dielectric function

$$K(\boldsymbol{r}, \boldsymbol{r}') = K(\boldsymbol{R} - \boldsymbol{R}', z, z') \tag{1}$$

(the frequency dependence is suppressed throughout), then the screened potential is

$$V(\boldsymbol{r}) = \int \mathrm{d}\boldsymbol{r}' \, K(\boldsymbol{r}, \boldsymbol{r}') U(\boldsymbol{r}'). \tag{2}$$

In the IBM geometry shown in figure 1, it is convenient to take the position vector $\boldsymbol{r} = (\boldsymbol{R}, z)$ and the wavevector $\boldsymbol{q} = (\boldsymbol{Q}, q_z)$. Defining the two-dimensional Fourier transform parallel to the interface by

$$F(\boldsymbol{Q}, z) = \int \mathrm{d}^2 \boldsymbol{R} \exp(-\mathrm{i}\boldsymbol{Q} \cdot \boldsymbol{R}) \, F(\boldsymbol{R}, z) \tag{3}$$

we obtain from equation (2)

$$V(\boldsymbol{Q}, z)=\frac{2\pi Ze}{Q}\int_0^\infty \mathrm{d}z'\, K(Q, z, z')\exp(-Q|z'-z_0|) \tag{4}$$

for an impressed Coulomb potential due to an impurity charge of strength Z at $\boldsymbol{r}_0=(0, 0, z_0)$ on the polar z axis. It is the longitudinal part of the dielectric tensor that enters our screening calculation, which, even in a constant magnetic field, is diagonal. Of course, the transverse part will have non-diagonal terms.

For a semi-infinite quantum plasma, described by the longitudinal dielectric function $\varepsilon(\boldsymbol{q})\equiv\varepsilon(\boldsymbol{Q}, q_z)$, we substitute $K(\boldsymbol{Q}, z, z')$ in the diagonal approximation [3] into equation (4) and obtain

$$\begin{aligned}V(Q, z<0)=\frac{2\pi Ze}{Q}\bigg(&\exp(-Q|z-z_0|)-\frac{\exp[Q(z-|z_0|)]}{1+Q\nu(Q,0)}\\&+\frac{2}{1+Q\nu(Q,0)}\int_0^\infty \mathrm{d}z'\, K_Q(z')\exp[-Q(|z'-z_0|-z)]\bigg)\end{aligned} \tag{5a}$$

and

$$\begin{aligned}V(Q, z>0)=\frac{2\pi Ze}{Q}\bigg[&\frac{Q\nu(Q,z)}{1+Q\nu(Q,0)}\bigg(\exp(-Q|z_0|)\\&-2\int_0^\infty \mathrm{d}z'\, K_Q(z')\exp(-Q|z'-z_0|)\bigg)\\&+\int_0^\infty \mathrm{d}z'\,[K_Q(z-z')+K_Q(z+z')]\exp(-Q|z'-z_0|)\bigg]\end{aligned} \tag{5b}$$

where

$$K_Q(z)\equiv\frac{1}{\pi}\int_0^\infty \mathrm{d}q_z\,\frac{\cos(q_z z)}{\varepsilon(\boldsymbol{q})} \tag{6a}$$

$$\nu(Q, z)\equiv\frac{2}{\pi}\int_0^\infty \mathrm{d}q_z\,\frac{\cos(q_z z)}{(q_z^2+Q^2)\varepsilon(\boldsymbol{q})}. \tag{6b}$$

Four cases are possible, depending on the signs of z and z_0. We Fourier transform equations (5) back to $\boldsymbol{R}$-space using the inverse of equation (3):

$$F(\boldsymbol{R})=\frac{1}{2\pi}\int_0^\infty \mathrm{d}Q\, QJ_0(QR)F(Q, z). \tag{7}$$

We obtain the following.

For case I ($z<0$, $z_0<0$),

$$V_{\mathrm{I}}(\boldsymbol{r})=\frac{Ze}{|\boldsymbol{r}-\boldsymbol{r}_0|}+Ze\int_0^\infty \mathrm{d}Q\, J_0(QR)\frac{Q\nu(Q,0)-1}{Q\nu(Q,0)+1}\exp(-Q|z+z_0|). \tag{8}$$

For case II ($z>0$, $z_0>0$),

$$V_{\mathrm{II}}(\boldsymbol{r})=Ze\int_0^\infty \mathrm{d}Q\, J_0(QR)Q\bigg(\nu(Q, z-z_0)+\nu(Q, z+z_0)-\frac{2Q\nu(Q,z)\nu(Q,z_0)}{1+Q\nu(Q,0)}\bigg). \tag{9}$$

For case III ($z<0$, $z_0>0$),

$$V_{\mathrm{III}}(\boldsymbol{r})=2Ze\int_0^\infty \mathrm{d}Q\, J_0(QR)\frac{Q\nu(Q,z_0)}{1+Q\nu(Q,0)}\exp(-Q|z|). \tag{10}$$

For case IV ($z > 0, z_0 < 0$),

$$V_{\rm IV}(\boldsymbol{r}) = 2Ze \int \mathrm{d}Q\, J_0(QR) Q\nu(Q, z) \exp(-Q|z_0|). \tag{11}$$

($J_0(x)$ is a Bessel function.) Further progress depends on determining the Q-dependence of the quantity $\nu(Q, z)$ defined by equation (6). For a dispersionless dielectric medium with $\varepsilon(\boldsymbol{q}) = \varepsilon_0$, we have $\nu(q, z) = \exp(-Q|z|)/Q\varepsilon_0$ and equations (8)–(11) become

$$V_{\rm I}(\boldsymbol{r}) = Ze\{1/|\boldsymbol{r} - \boldsymbol{r}_0| + [(1 - \varepsilon_0)/(1 + \varepsilon_0)](1/|\boldsymbol{r} - \boldsymbol{r}_0'|)\} \tag{12}$$

$$V_{\rm II}(\boldsymbol{r}) = \frac{Ze}{\varepsilon_0}\{1/|\boldsymbol{r} - \boldsymbol{r}_0| - [(1 - \varepsilon_0)/(1 + \varepsilon_0)](1/|\boldsymbol{r} - \boldsymbol{r}_0'|)\} \tag{13}$$

$$V_{\rm III}(\boldsymbol{r}) = [2Ze/(1 + \varepsilon_0)](1/|\boldsymbol{r} - \boldsymbol{r}_0|) = V_{\rm IV}(\boldsymbol{r}). \tag{14}$$

Here we have introduced an image charge at $\boldsymbol{r}_0' = (0, 0, -z_0)$.

In many situations, including the electron gas in the HFQL in which all electrons are confined to the lowest Landau state, the bulk dielectric function $\varepsilon(\boldsymbol{q})$ depends only weakly on Q for small values of this wavenumber and we have [9]

$$\nu(Q, z) = \frac{2}{\pi}\int_0^\infty \mathrm{d}q_z \frac{\cos(q_z z)}{q_z^2 \varepsilon(0, q_z)} + \mathrm{O}(Q^2). \tag{15}$$

With the exception of case II, we can take advantage of equation (15) to estimate $V(\boldsymbol{r})$ for large values of $|z|$ and $|z_0|$. Let $\nu(z)$ denote the first term in equation (15) and consider the second (image) term in equation (8). Since $|z + z_0|$ is large, by Watson's lemma (see, e.g., [11]) the asymptotic behaviour of the integral in equation (8) is dominated by the small-Q behaviour of the integral. After a little algebra, equation (8) yields

$$V_{\rm I}(\boldsymbol{r}) \simeq Ze/|\boldsymbol{r} - \boldsymbol{r}_0| - Ze/|\boldsymbol{r} - \boldsymbol{r}_0'| + 2Ze\nu(0)|z + z_0|/|\boldsymbol{r} - \boldsymbol{r}_0|^3. \tag{16}$$

Applying this procedure to equation (10) and assuming that z is large and negative, we obtain

$$V_{\rm III}(\boldsymbol{r}) \simeq 2(Ze/r^3)|z|. \tag{17}$$

Similarly, for z_0 large and negative, equation (11) yields

$$V_{\rm IV}(\boldsymbol{r}) \simeq 2Ze\nu(z)|z_0|/|R^2 + z_0^2|^{3/2}. \tag{18}$$

This shows that, for example, in the HFQL the screened potential in the half-space *opposite* to the charge decays in the transverse direction as R^{-3}. The possible existence of the Friedel–Kohn wiggle requires an analysis of $\nu(z)$ which we now consider.

In the HFQL for small values of Q, we have [8]

$$\varepsilon(0, q_z) \simeq \varepsilon_0 - (4\pi e^2/q_z^3)\rho_0(m^*/2\hbar^2\zeta)^{1/2} \ln|(q_z - 2q_{\rm F})/(q_z + 2q_{\rm F})|. \tag{19}$$

Here ρ_0 is the electron number density in the bulk medium, $\zeta = \hbar^2 q_{\rm F}^2/2m^*$ is the magnetic field-dependent chemical potential and m^* is the effective mass of the electron. For a fixed number of electrons, the chemical potential has to change with magnetic field since the degeneracy of the Landau levels varies with the magnetic field strength. (For

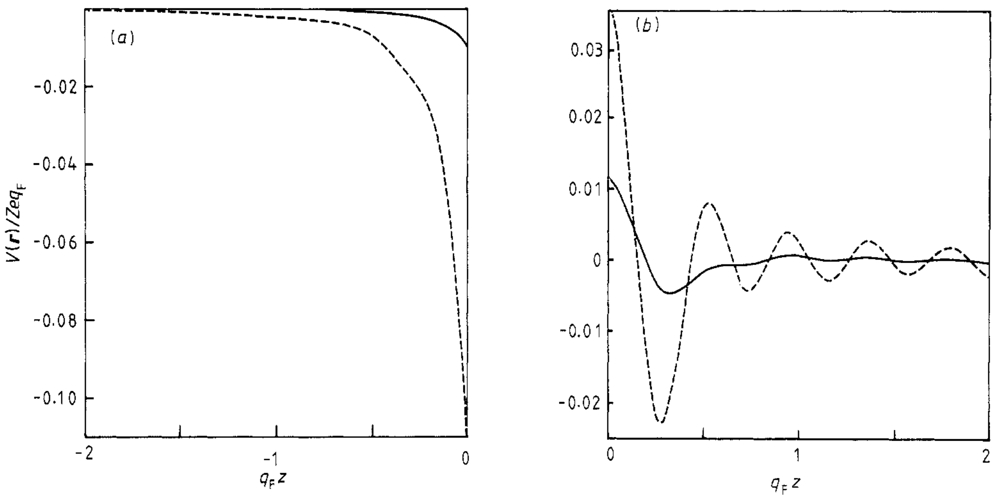

Figure 2. Plot of the screened Coulomb potential $V(R = 0, z)/Zeq_F$ given by equations (10) and (11), as a function of $q_F z$, when (*a*) the Coulomb impurity lies within the plasma on the polar z axis at a distance $z_0 = 2q_F^{-1}$ and the field point coordinate lies *outside* in the vacuum region and (*b*) the impurity lies in the vacuum at $z_0 = -q_F^{-1}$ on the polar z axis and the field point is *inside* the plasma: ———, for a background dielectric constant $\varepsilon_0 = 10.94$; - - - for $\varepsilon_0 = 1$.

a full discussion of the magnetic field dependence of the chemical potential, see § III of [12]). Substituting equation (19) into equation (15) and setting $Q = 0$, we obtain

$$\nu(z) = \frac{2}{\pi q_F \varepsilon_0} \int_0^\infty dx \frac{x \cos(q_F z x)}{x^3 + (\lambda/\varepsilon_0) \ln|(x+2)/(x-2)|} \tag{20}$$

where $\lambda \equiv 2\pi e^2 \varepsilon_0 / q_F^2 \zeta$. Since the integrand is analytic at $x = 0$, the leading asymptotic behaviour must be due to the logarithmic singularity of the integrand at $x = 2$. After some algebra, we obtain (see the Appendix for details)

$$\nu(z) = -4\cos(2q_F)z/\{\lambda q_F^2 |z| [\ln(q_F|z|)]^2\} + O[1/\ln(q_F|z|)]^3 \tag{21}$$

which shows that the leading order term for $\nu(z)$ is independent of the background dielectric constant ε_0. Employing equation (21) in equations (17) and (18), we obtain for both $|z|$ and $|z_0|$ large

$$V_{III}(r) \simeq -8Ze|z|\cos(2q_F z_0)/\{\lambda q_F^2 r^3 |z_0| [\ln(q_F|z_0|)]^2\} \tag{22}$$

and

$$V_{IV}(r) \simeq -8Ze|z_0|\cos(2q_F z)/\{\lambda q_F^2 |z| (R^2 + z_0^2)^{3/2} [\ln(q_F|z|)]^2\}. \tag{23}$$

In this paper, we offer a numerical RPA evaluation of V_{III} and V_{IV} when the test charges are on opposite sides of the surface, displaying the role of the background dielectric constant appropriate to semiconductors in which the quantum strong-magnetic-field limit can be achieved. The appropriate bulk dielectric function $\varepsilon(q)$ for such a full static shielding analysis in the quantum strong-field limit has been given in [8]. Employing this in equation (6*b*) and then evaluating the integrals in equations (10) and (11), we obtain numerical results for $V(r)$ shown in figure 2 for $R = 0$ and values of

$q_F|z_0| \gg 1$ so that the role of the omitted quantum interference terms may be expected to be relatively unimportant in accordance with the results in [6]. In the calculations, we take the magnetic field $H = 10^5$ G and for GaAs we use an electron effective mass $m^* = 0.0665m_e$, bulk density $\rho_0 = 10^{16}$ cm^{-3}, $\varepsilon_0 = 10.94$, Fermi wavenumber $q_F = 0.0013$ Å^{-1}, chemical potential $\zeta = 9.65 \times 10^{-5}$ eV and cyclotron frequency $\omega_c = 2.65 \times 10^{13}$ s^{-1}. Since $\hbar\omega_c/\zeta > 1$ for these values, the quantum strong-field limit is achieved, and only the lowest Landau eigenstate is occupied. We note the Friedel–Kohn wiggle in the RPA shielded potential shown as a function of distance from the boundary (for $R = 0$) in all parts of figure 2(b). In addition to the full curves which show $V(z)$ for GaAs with $\varepsilon_0 = 10.94$, the corresponding results for $\varepsilon_0 = 1$ (but with all other numbers characteristic of GaAs the same as indicated above) are shown as broken curves to emphasise the importance of incorporating the proper value of ε_0. The case of the source lying inside and the field point outside in the vacuum region (case III) differs substantially from the case when the source and field points are interchanged (case IV). In the former case, there is no charge present in the outside region and therefore no Friedel–Kohn wiggle of density or potential is possible. This is demonstrated numerically in figure 2(a) and analytically by the long-distance asymptotic result in equation (22). However, for case IV the density perturbation of the inside region is capable of sustaining a Friedel–Kohn wiggle which is demonstrated in figure 2(b) and also for large distances by equation (23). It is to be noted that under the high-magnetic-field conditions which we are concerned with here (the quantum strong-field limit in which all electrons are confined to the lowest Landau state), the Friedel–Kohn wiggle contribution to the shielded potential dies off with distance as $\{q_F|z|[\ln(q_F|z|)]^2\}^{-1}$ for source-field displacements perpendicular to the surface. On the contrary, for displacements parallel to the surface and perpendicular to the magnetic field direction, the potential decays with distance as $(R^2 + z_0^2)^{-3/2}$, a pure power-law behaviour.

Appendix

In this Appendix, we evaluate the integral

$$\nu(z) = \frac{2}{\pi q_F \varepsilon_0} \int_0^\infty \mathrm{d}x \frac{x\cos(q_F z x)}{x^3 + (\lambda/\varepsilon_0)\ln|(x+2)/(x-2)|} \tag{A1}$$

for large values of $q_F z$. The integrand is analytic at $x = 0$ so that, for large $q_F z$, the integral is dominated by the logarithmic singularity at $x = 2$. Hence there will be oscillations $\sim\cos(2q_F z)$ within $\nu(z)$.

Consider the integral

$$\varphi(\alpha) = \int_0^\infty \mathrm{d}x \frac{x\cos(\alpha x)}{x^3 + b\ln|(x+a)/(x-a)|} \tag{A2}$$

where a, b and α are all real quantities. We now calculate φ for $\alpha \to \infty$. Since the integrand decays exponentially in the upper half of the complex z plane, we can deform the contour as shown in figure A1. Since the integrand decays exponentially on C_3 and C_4, we obtain

$$\varphi(\alpha) = \mathrm{Re}\left(\int_{C_0+C_1} \mathrm{d}z \frac{z\exp(\mathrm{i}\alpha z)}{z^3 + b\ln|(a+z)/(a-z)|} + \int_{C_2} \mathrm{d}z \frac{z\exp(\mathrm{i}\alpha z)}{z^3 + b\ln|(z+a)/(z-a)|}\right) \tag{A3}$$

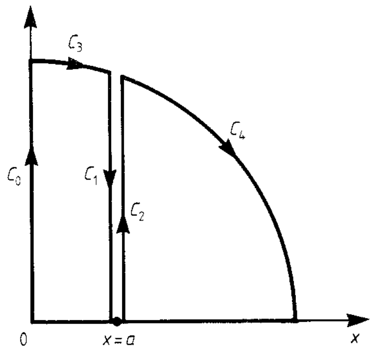

Figure A1. Contour of integration of equation (A3).

where $\mathrm{Re}(x)$ stands for the real part of x. If we set $z = \mathrm{i}u$ on C_0, the resulting integral is purely imaginary and so does not contribute to $\varphi(\alpha)$. Let $z = a + \mathrm{i}u$ on C_1 and C_2. Therefore, we obtain

$$\varphi(\alpha) = -\pi b\,\mathrm{Re}\left(\exp(\mathrm{i}\alpha a)\int_0^\infty \mathrm{d}u\,\frac{(a+\mathrm{i}u)\exp(-\alpha u)}{[(a+\mathrm{i}u)^3 + b\ln(2a+\mathrm{i}u) - b\ln u]^2 + (\pi b/2)^2}\right). \tag{A4}$$

By Watson's lemma [11], the dominant behaviour as $\alpha \to \infty$ is

$$\varphi(\alpha) \simeq -\pi ab\,\mathrm{Re}\left(\exp(\mathrm{i}\alpha a)\int_0^\infty \mathrm{d}u\,\frac{\exp(-\alpha u)}{(a^3 + b\ln 2a - b\ln u)^2 + (\pi b/2)^2}\right). \tag{A5a}$$

That is

$$\varphi(\alpha) \simeq -\frac{\pi a}{b}\cos(\alpha a)\int_0^\infty \mathrm{d}u\,\frac{\exp(-\alpha u)}{(\ln u)^2}. \tag{A5b}$$

But

$$\int_0^\infty \mathrm{d}u\,\frac{u^s\exp(-\alpha u)}{(-\ln u)^\beta} \simeq \frac{\Gamma(s+1)}{\alpha^{s+1}}\left[\frac{1}{(\ln\alpha)^\beta} + \mathrm{O}\left(\frac{1}{(\ln\alpha)^{\beta+1}}\right)\right]. \tag{A6}$$

Hence

$$\varphi(\alpha) = -[\pi a\cos(\alpha a)/b\alpha(\ln\alpha)^2][1 + \mathrm{O}(1/\ln\alpha)]. \tag{A7}$$

From equation (A7), we obtain the asymptotic value of $\nu(z)$ in equation (A1). This large-distance behaviour is given in equation (21).

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